Extra Practice Problems 6

More Fun With Friends and Strangers

(From the Fall 2013 midterm exam)

Suppose you have a 17-clique (that is, an undirected graph with 17 nodes where there's an edge between each pair of nodes) where each edge is colored one of *three* different colors (say, red, green, and blue). Prove that regardless of how the 17-clique is colored, it must contain a blue triangle, a red triangle, or a green triangle. *(Hint: Use the theorem on friends and strangers.)*

Bijections and Induction

(From the Fall 2014 midterm exam)

Let $f : \mathbb{N} \to \mathbb{N}$ be a function. We'll say that f is ***linearly bounded*** if $f(n) \le n$ for all $n \in \mathbb{N}$.

Prove that if $f : \mathbb{N} \to \mathbb{N}$ is linearly bounded and is a bijection, then $f(n) = n$ for all $n \in \mathbb{N}$. *(Hint: You might find induction useful here.)*

A good question to ponder: is this result still true if we replace $\mathbb{N}$ with $\mathbb{Z}$?

Odd Rational Numbers

On Problem Set Three, you explored the binary relation ~ over $\mathbb{R}$ defined as follows:

$$x \sim y \quad \text{if} \quad y - x \text{ is an odd rational number.}$$

Here, an odd rational number is a rational number that can be written with an odd denominator.

Prove that every element of $[\sqrt{2}]_\sim$ is irrational.

Long Paths

(From the Fall 2016 midterm exam)

Let $G = (V, E)$ be a graph where every node has degree at least k for some $k \ge 1$. Let P be a simple path in G that has length less than k. Prove that P is ***not*** the longest simple path in G.